Orchestrating Whole Group Discourse to Mediate Mathematical Meaning

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Abstract

The most common pattern of classroom discourse follows a three-part exchange of teacher initiation, student response, and teacher evaluation or follow-up (IRE/IRF) (Cazden, 2001). Although sometimes described as encouraging illusory understanding (Lemke, 1990), triadic exchanges can mediate meaning (Nassaji & Wells, 2000). This paper focuses on one case from a study of discursive practices of seven middle grades teachers identified for their expertise in mathematics instruction. The central result of the study was the development of a model to explain how teachers use discourse to mediate mathematical meaning in whole group instruction. Drawing on the model for analysis, thick descriptions of one teacher's skillful orchestration of triadic exchanges that enhance student understanding of mathematics are presented.

Orchestrating Whole Group Discourse to Mediate Mathematical Meaning

Introduction

Recent reform efforts have identified communication as essential to the teaching and learning of mathematics (NCTM, 2000). However, the mere presence of talk does not ensure that thinking and understanding follow. Research has demonstrated that the *quality* and *type* of discourse greatly affect its potential for promoting conceptual understanding (Chapin, O'Connor, & Anderson, 2003; Kazemi & Stipek, 2001; Lampert & Blunk, 1998; Whitenack & Yackel, 2002). The teacher's role is vital in the orchestration of discourse (Barnes, 1992; Confrey, 1995) and has been underrepresented in the literature (Chazan & Ball, 1995; Jaworski, 1997). This paper focuses on one case from a larger study of discursive practices of seven middle grades mathematics teachers identified for their expertise in mathematics instruction (Truxaw & De Franco, 2004). The question addressed is: *How does an expert middle grades mathematics teacher orchestrate whole group discourse to mediate mathematical meaning?*

Background

Social constructivism, with its contention that higher mental functions derive from social interaction, provides a meaningful framework for analysis and discussion of discourse as a mediating tool in the learning-teaching process. Verbal exchanges between more mature and less mature participants may develop back and forth processes from thought to word and from word to thought that allow learners to move beyond what would be easy for them to grasp on their own (Vygotsky, 1986, 2002). For example, in mathematics classrooms, conversations between the teacher and the student may provide semiotic mediation that may, in turn, promote mathematical meaning-making. When examining language as a mediator of meaning, it is useful to consider the two main functions of communication—that is, "to produce a maximally accurate transmission of a message" and "to create a new message in the course of the transmission" (Lotman, 2000, p. 68), characterized as *univocal* and *dialogic* discourse, respectively (Wertsch, 1998).

In addition to functions of discourse, researchers have identified basic structures found particularly within classrooms (e.g., Berry, 1981; Halliday, 1978; Mehan, 1985; Sinclair & Coulthard, 1975; Wells, 1999). For example, classroom discourse has been parsed according to the following categories: a move, an exchange, a sequence, and an episode (Wells, 1999). The move, exemplified by a question or an answer from one speaker, is identified as the smallest building block. An exchange, made up of two or more moves, occurs between speakers. Typically, the teacher initiates an exchange, with the student responding, and the teacher following-up with either an evaluation or some sort of feedback to the student's response. Exchanges are categorized as either nuclear (i.e., can stand alone) or bound (i.e., dependent upon or embedded within previous exchanges). The sequence is the unit that contains a single nuclear exchange and any exchanges that are bound to it. Finally, an episode is comprised of all the sequences that are necessary to complete an activity.

The most common pattern of classroom discourse follows the three-part exchange of teacher initiation, student response, and teacher evaluation or follow-up (i.e., IRE or IRF) (Cazden, 1988, 2001; Coulthard & Brazil, 1981; Mehan, 1985). The triadic structure has been criticized as encouraging "illusory participation"—that is, participation that is "high on quantity, low on quality"—because "it gives the teacher almost total control of classroom dialogue and social interaction" (Lemke, 1990, p. 168). However, Nassaji and Wells (2000) found that, even within inquiry-style instruction, triadic dialogue was the dominant structure and, therefore, important to take into account when examining classroom discourse. Further, it was noted that within triadic exchanges, the teacher's initiation and follow-up moves influence the function of the discourse. For example, when moves are used to evaluate a student's response, the intention of the discourse is likely to tend toward transmitting meaning (i.e., tending toward univocal). In contrast, questions that invite students to contribute ideas that might change or modify a discussion are more likely to be associated with dialogic discourse (Wells, 1999; Wertsch, 1998).

Along with social constructivist theory and classroom discourse research, socio-linguistics provides another lens for viewing and making sense of the flow, forms, functions, and intentions of classroom talk. For example, line-by-line coding strategies used in this study were adapted from Wells (1999) to enhance the analysis of classroom discourse transcripts. Additionally, forms of talk and verbal assessment within whole group discussion were identified (Truxaw & DeFranco, 2004), including:

monologic talk (i.e., involves one speaker—usually the teacher—with no expectation of verbal response), *leading talk* (i.e., occurs when the teacher controls the verbal exchanges, leading students toward the teacher's point of view), *exploratory talk* (i.e., speaking without answers fully intact, analogous to preliminary drafts in writing) (Cazden, 2001), *accountable talk* (i.e., talk that requires accountability to accurate and appropriate knowledge, to rigorous standards of reasoning, and to the learning community) (Michaels, O'Connor, Hall, & Resnick, 2002), *inert assessment* [IA] (i.e., assessment that does not incorporate students' understanding into subsequent moves, but rather, guides instruction by keeping the flow and function relatively constant), and *generative assessment* [GA] (i.e., assessment that mediates discourse to promote students' active monitoring and regulation of thinking about the mathematics being taught).

Methods and Procedures

This paper focuses on one episode within one case from within a larger study. The participants in the larger study were a purposive sample of seven middle grades mathematics teachers (grades 4 – 8) who were identified as having characteristics indicative of expertise (Darling-Hammond, 2000), including representatives from three specific groups: teachers who had achieved *National Board for Professional Teaching Standards* [NBPTS] certification in Early Adolescent Mathematics, recipients of the *Presidential Award for Excellence in Mathematics and Science Teaching* [PAEMST], and teachers recommended by university faculty. Discursive practices of one of these participants, Jacob (a pseudonym), an eighth grade mathematics teacher, are highlighted in this paper. Jacob's background included several indicators of expertise: 35 years of public school teaching experience, certification in secondary mathematics (grades 7 – 12), NBPTS certification, advanced certification in educational technology, bachelor's degree in economics, master's degree in mathematics education, and university-level mathematics education teaching experience.

The data were collected via semi-structured interviews, classroom observations, field notes, audiotapes, and videotapes. Grounded theory methodology (Strauss & Corbin, 1990), multiple-case study design (Stake, 1995; Yin, 1994), and sociolinguistic tools (Wells, 1999) were applied within a social constructivist framework (Vygotsky, 1986, 2002; Wertsch, 1991, 1998) to analyze the discourse. The transcripts from the classroom observations were coded on several levels—for example, line-by-line coding of moves was accomplished using schemes adapted from Wells (1999) (e.g., see Table 1) and sequences were coded using strategies developed during a pilot study. Next, individual *sequence maps* (i.e., diagrams representing the flow of forms of talk and verbal assessment within a sequence—see figure 1) were created by applying the coded data to a preliminary graphic model of classroom discourse. Maps and coded transcripts were inspected, compared, adapted, and synthesized to develop an overall model of the flow of classroom discourse.

Table 1. Example of Line-by-Line Coding Adapted from Wells (1999).

Map #	Line #	Seq #	Who	Text	K1 K2	Exch	Move (I-R-F)	Prospect-iveness	Function
3	49	4	T	What are the factors of six? B9?	K1	Nuclear	I	Demand	Req inf
4	50	4	B9	2, 3, 1, and 6	K2	Dep	R	Give	Inform
5	51	4	T	Okay.	K1	Dep	F	Acknow	Acknow

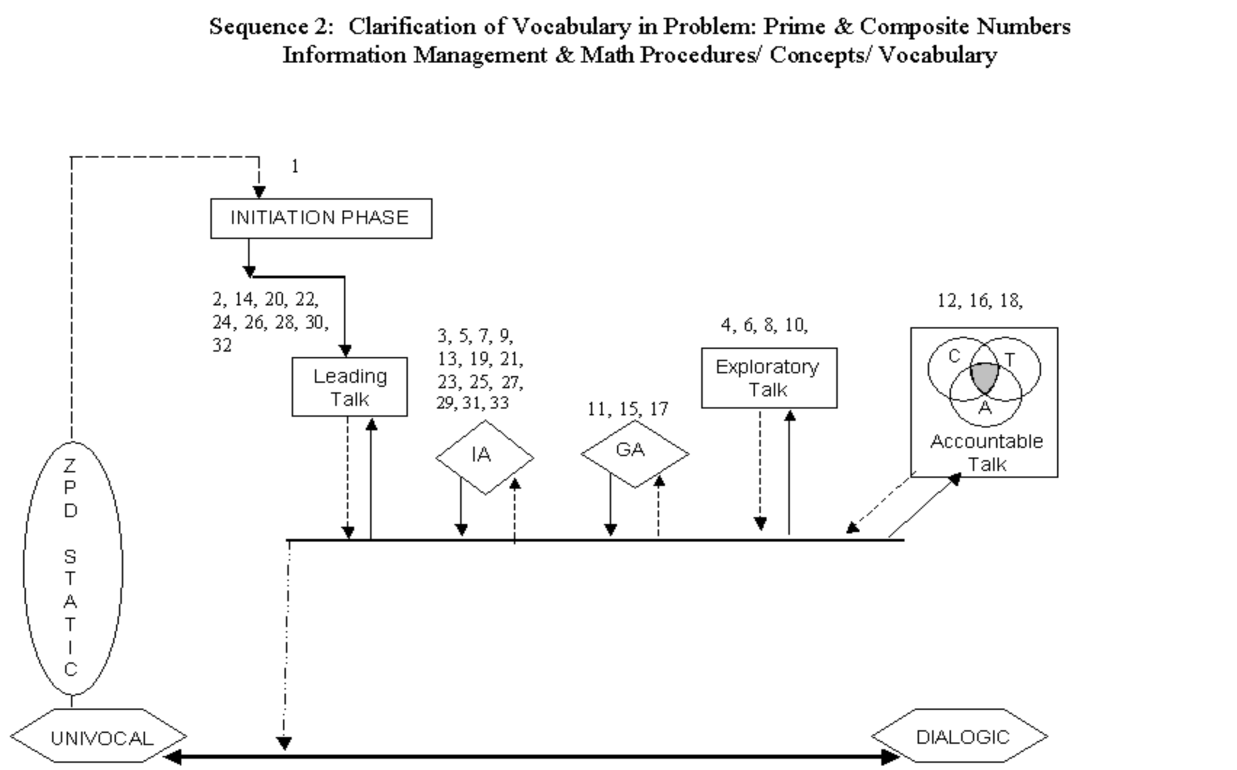
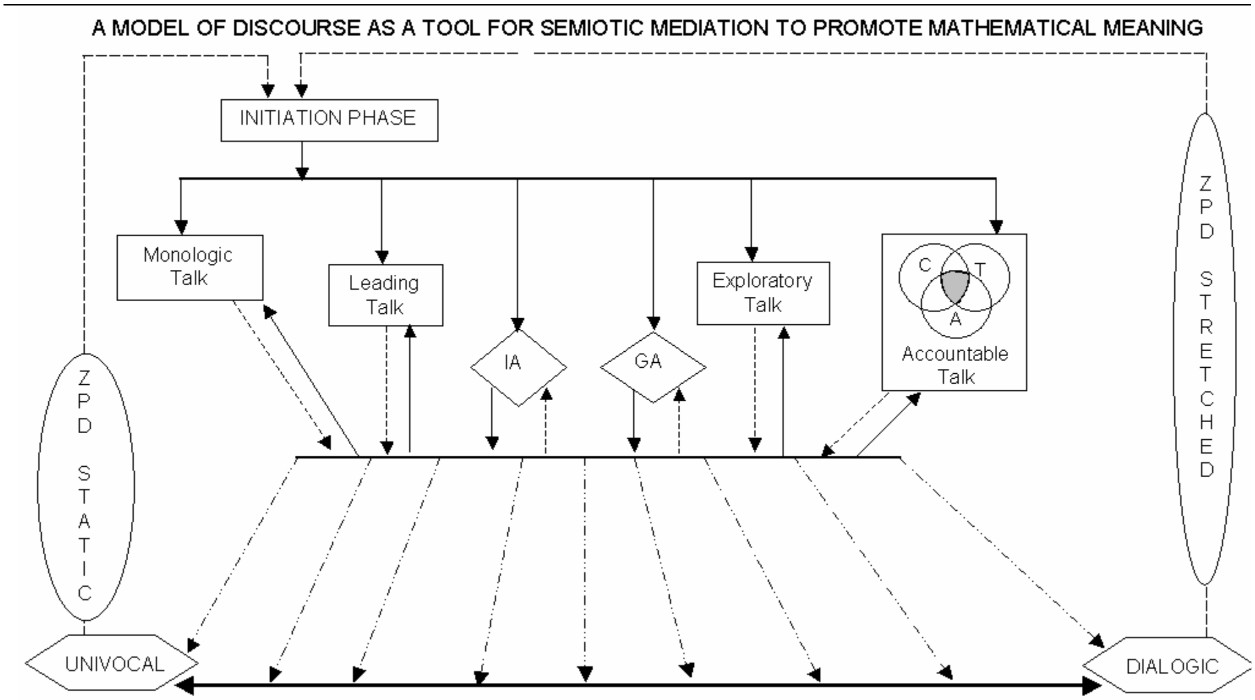


Figure 1. Example of a sequence map representing the flow of particular forms of talk and verbal assessment. The numbers represent individual verbal moves in sequential order.

The central result of the larger study was the development of a dynamic model to explain how teachers use discourse to mediate mathematical meaning in whole group instruction (see Figure 2) (Truxaw & De Franco, 2004). The model provided a template for mapping the flow of forms of talk (i.e., monologic, leading, exploratory, and accountable) and forms of verbal assessment (i.e., IA and GA), along with tendencies toward univocal (transmitting meaning) or dialogic (generating meaning) function. Through its focus on the interactions of talk and verbal assessment and their relationships toward univocal and dialogic tendencies, the model supplied indicators to whether meaning was being conveyed or it was being constructed. Building a model of the flow of discourse provided a theoretical foundation and analytic tools to investigate how an expert mathematics teacher orchestrates whole group discourse to mediate mathematical meaning.



GA = Generative Assessment; IA = Inert Assessment; C = Accountable to Community; A = Accountable to Accuracy; T = Accountable to Thinking

The model begins with an initiation phase, from which the discourse moves to a category of talk or verbal assessment. The type of verbal assessment is critical to the movement of discourse. IA provides little or no semiotic mediation and, therefore, tends to maintain existing discursive functions. GA promotes semiotic mediation and discourse is more likely to progress toward dialogic function.

Figure 2. Model of discourse to mediate mathematical meaning.

To unpack how discourse could be orchestrated to mediate mathematical meaning, fine-grained analysis of the individual cases was undertaken. Certain sequences and instructional episodes were identified as potentially informative—with particular focus on sequences that included the following: evidence of discourse that tended toward univocal function; evidence of discourse that tended toward dialogic function; and evidence of discourse that tended toward dialogic function, but then shifted back toward univocal function. Multi-level analysis of sequence maps, lesson transcripts, interview transcripts, and field notes of these selected episodes and sequences was completed. These analyses resulted in the development of models of teaching that demonstrated greater or lesser tendencies toward meaning-making. This paper reports on findings related to the analysis of one teaching/learning episode from Jacob's eighth grade mathematics class that was representative of an *inductive teaching model* and also demonstrated tendencies toward promoting meaning-making.

Results and Discussion

Overview of an Inductive Teaching/ Learning Episode

The inductive teaching model (see Figure 3) was built from a learning episode that consisted of four sequences in one of Jacob's eighth grade mathematics classes. A problem was introduced in sequence one (i.e., "What is the sum of the reciprocals of the prime or composite factors of 28?"), establishing a frame of reference (see Figure 3-A). In sequence two, common understanding of key terms was developed (e.g., prime and composite) (see Figure 3-B-1), while in sequence, three the problem was investigated in small groups and a solution was presented by a student (see Figure 3-B-2&3). By consensus, the class agreed that the sum of the reciprocals of the prime and composite factors of 28 equaled 1, which provided a new basis for meaning-making (See Figure 3-C). Within the first three sequences, all four forms of talk and both IA and GA were used, but the overall function of the discourse was univocal in nature.

Although one might imagine that the learning episode would be complete with the presentation and acceptance of a solution, instead, the first three sequences served as a springboard for sequence four. The fourth sequence built from the first three, using the problem as a frame of reference for developing, testing and revising hypotheses; exploring connections between the problem's solution and other concepts (e.g., abundant numbers, deficient numbers, and perfect numbers); constructing revised frames of reference; and demonstrating students' understanding related to the original problem and the revised hypotheses. The cyclic nature of the discourse (i.e., recursively establishing common understanding, exploring, conjecturing, testing, and revising hypotheses) was used to progressively build new meaning (see Figure 3D-3G). The discourse in the fourth sequence was particularly complex—including multiple instances of leading, exploratory, and accountable talk and both IA and GA. Also of note is that accountable talk and GA occurred more frequently in sequence four than they had in the previous three sequences. The method of instructive was predominantly inductive—that is, moving from specific cases toward more general hypotheses and rules. The discourse in the learning episode (i.e., sequences 1– 4) moved from relatively univocal (while building common understanding) to relatively dialogic (as the common themes were used to build new meaning).

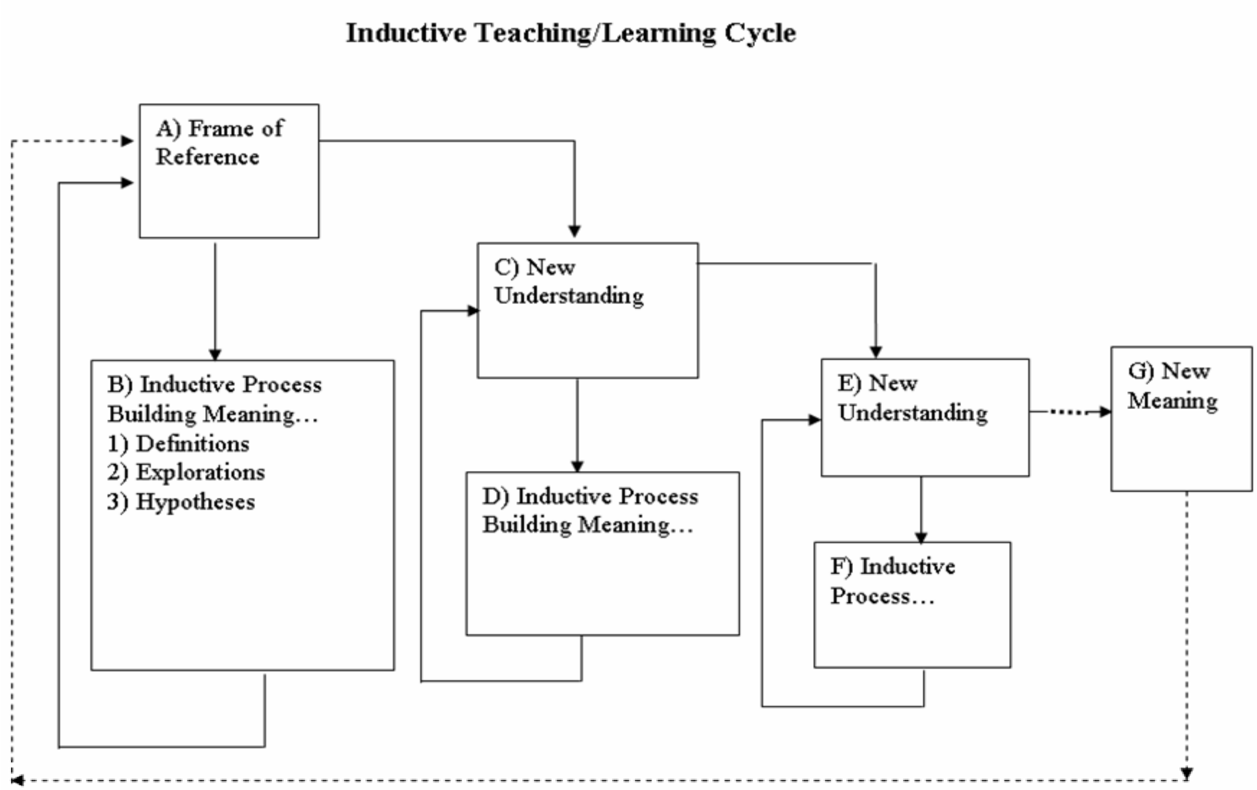


Figure 3. Inductive teaching model.

Taking a Closer Look at the Episode

The fine-grained analysis of this teaching/learning episode (that resulted in the development of the inductive teaching model) included re-examination of the coded classroom transcripts, the sequence maps, pre- and post-observation interview transcripts, and field notes. These provided evidence for considering how the content, the flow, and the intent might influence the functions and outcomes of the discourse.

Sequence one revisited. In the first sequence, classic triadic exchange structure (IRE/IRF), as represented by monologic talk, leading talk and IA, was used to introduce the problem (i.e., *What is the sum of the reciprocals of the prime or composite factors of 28?*) that would serve as a frame of reference for the following three sequences. The problem had derived from one that Jacob's students had been given in a mathematics problem-solving competition the previous week—that is, *Find the sum of the reciprocals of all the factors of 28* (i.e., 1/1 + ½ + ¼ + 1/7 + 1/14 + 1/28 = 48/28 = 2). In an excerpt from Jacob's post-observation interview, he described how his own curiosity about the problems from the math competition led him to the "guided discovery" teaching/learning episode reported:

> "I particularly liked the first problem and I was surprised at the result, [and] said, 'Wow! If you take one off of here, you get one. Is that always going to be true—the reciprocals of the factors?' And I happened to choose six as the next number I tried and it worked again … I said, *[inaudible]* … we've got some funny … it's a *perfect number*… then I said, oh, yeah, 28's a perfect number too. What if it's not a perfect number? I tried another number and it didn't work. And I realized, you know, it only works for perfect numbers. And I thought about how it made it sense. And wouldn't it be neat if kids could come to that … themselves? So…I tried to massage the problem to make it to be that … and thought it might lead to a nice discussion about it …"

Jacob added, "It's so important that rather than just go into an answer key, and you can't always do it, but whenever it's possible, that you go through the experience that your kids have." Jacob's process of working through the mathematics and *making his own discoveries* helped him to facilitate guided discovery with his students, as will be shown in the explanations of sequences two, three, and four.

Sequence two revisited. Like the first sequence, the discourse in the second sequence was characterized by triadic exchanges between Jacob and the students. However, this sequence was more complex than the first, including examples of leading, exploratory, and accountable talk and both IA and

GA. The overall purpose of the sequence seemed to be the development of a common understanding of the vocabulary necessary for the problem—that is, prime and composite numbers. To establish this common language, Jacob did not simply define the words; rather, he allowed students to express their own understanding, using both IA and GA to promote movement toward accurate and appropriate knowledge. For example, when a student offered an incomplete definition for prime numbers, "Numbers that can only be divided by one and itself," he asked for agreement from the class. When students agreed, he continued, facilitating examples and counterexamples, until a student provided the missing piece: "It has exactly two factors." Instead of simply accepting this, he used GA to probe, "How is that different?" The student provided an example, "Well, because one can divide itself, but it only has one factor." Overall, discourse tended to be univocal in that its purpose was to convey accurate meaning about the vocabulary; however, there was some tendency toward dialogic function since students' voices were key parts of the meanings that were represented and, additionally, justifications and explanations were included in the dialogue.

Sequence three revisited. The third sequence included whole group discourse (that was fully coded) and small group interactions (that were not coded line-by-line, but were described within field notes). Most of the verbal discourse during this sequence diverged from triadic structure—with a more open-ended, exploratory stance. During the small group interactions, Jacob circulated throughout the room asking predominantly general, inert questions like, "How are you doing?" Although this type of question fell into the category of IA, Jacob indicated (post-observation interview) that its purpose was to ensure that the students continued to explore the mathematical relationships inherent in the problem, as well as to provide a foundation for other assessments that would eventually lead toward the development of an understanding of the mathematics. During the small group work, Jacob listened and observed, identifying particular students who might need assistance and also students whom he might call on during large group discussion to provoke meaningful discourse (post-observation interview). When the whole group reconvened, Jacob asked a student to come to the board to demonstrate his strategies and share his solution. By consensus, the class agreed that the sum of the reciprocals of the prime and composite factors of 28 equaled 1. These first three sequences provided common understanding that served as a springboard from which sequence four was built.

Sequence four revisited. Jacob's fourth sequence was the most complex from among the 120 sequences that were mapped in the larger study. It included 6 leading talk moves, 10 exploratory talk moves, 28 accountable talk moves, 26 IA moves, and 20 GA moves. The discourse in this sequence moved back and forth between whole group discourse (with predominantly triadic structure) and small group talk. During this sequence students were allowed to serve in the role of the primary knower[1], the teacher modeled metacognitive reasoning (Flavell, 1979), and new meaning was generated. Jacob shifted his role throughout in the discourse—sometimes "stepping in" to serve as a participant in the discussion and sometimes "stepping out" to serve as a commentator who facilitated and clarified rules, procedures, concepts, arguments, and classroom norms (Rittenhouse, 1998). In sequence four, Jacob orchestrated the introduction of a hypothesis related to the problem that was introduced in sequence one.

Jacob	That's sort surprising that it would actually be one. I wonder if that's always true. I'm going to try another. I'm going to try six. Somebody said something about six. What are the factors of six? Bruce?
Bruce[2]	2, 3, 1, and 6
Jacob	Okay. *[Writes factors on board.]* So, again, what we said was, we're going to only use the prime and composite factors, right? So we'll throw out this one. *[Crosses out the 1.]* So we have ½ plus 1/3 plus 1/6, right? And I like what David did, *[references student work from the previous sequence]* which was, he made a same denominator. *[Shows 3/6 + 2/6 + 1/6 on board.]* And what do we get?
Lindsey	6/6
Jacob	Which is **one**! Woe! So what should we call this? Should we call this the '*Hankins* Property' or what? *[using David's last name]* You want credit for it, David?
David	Definitely.
Jacob	Definitely. Well, I wonder if this always works. It looks kind of neat. We've seen two examples now where it works. I'm sort of surprised…I don't know why it would work, but it seems to work.
Jacob	Would you guys check it out? Would you each take some other number and check it to see if, in fact, it does work?

[1] *Primary Knower* – The person who "knows" the information and imparts it; *Secondary Knower* – The person to whom the information is imparted (Berry, 1981)

[2] All names are pseudonyms.

The students worked cooperatively in small groups at tables, testing out numbers. During this time, Jacob wrote the "Hankins Hypothesis" on the board—that is, *Sum of reciprocals of prime and composite factors of a number will always be one*. Jacob then circulated around the room, listening and asking questions. For example, when a student said, "It doesn't work for primes," Jacob asked him to think about why that might be so. After calling the class's attention back to the whole group, Jacob asked, "Okay, so what did you discover?" As the students shared their "discoveries," Jacob orchestrated the addition of two potential "corollaries" to the "Hankins Hypothesis" (that he named after the students who uncovered them)—that is, the *Vargas Corollary* said that *the hypothesis doesn't work for primes* and the *O'Connor Corollary* said that *the hypothesis doesn't work for perfect cubes*. Additional numbers for which the *Hankins Hypothesis* didn't work were shared, until a student said, "It didn't work for 36, which is an *abundant number*."

Jacob	Woe! A what? *[dramatically]*
David	An abundant number.
Jacob	An **abundant** number! What is an abundant number?
David	When the factors of the number add up to more than the number itself.
Jacob	Have you guys heard of that before? An **abundant** number?
Students	*[A couple of students indicate that they have]*
Jacob	And do you know what it's called when a number, the sum of the factors, add up to something less than the number?
Kohei	A non-abundant number?
Jacob	Somebody … A non-abundant number … somebody … I heard it out there.
Lindsey	Deficient.
Jacob	Deficient. So, a **deficient** number and an **abundant** number…

The class then briefly discussed the meanings of abundant and deficient numbers. Then a student made a connection between the new knowledge of deficient numbers and the *Hankins Hypothesis*.

Jacob	And, Kohei, you have a big smile on your face.
Kohei	It doesn't work for deficient numbers.
Jacob	It doesn't work for deficient numbers. Hmmm. Daniel.
Daniel	Uh, you said, 'abundandant.' *[Refers to spelling mistake on board.]*

Jacob   Abundantant – I'm just making up a new word. So, huh? It doesn't work for abundant… Is there anything that we call it when the sum of the factors of a number itself **equals** the number?

David   Perfect numbers?

Jacob   It's called a **perfect** number. It's called a **perfect** number.

Jacob asked students how many had heard of perfect numbers, and then asked if anyone knew any perfect numbers. Students made connections between the original problem (from sequence one) and perfect numbers.

Jacob   … Perfect number. Well, anybody know any perfect numbers? Daniel?

Daniel   Six.

[Recall that the number 6 worked for the "Hankins Hypothesis."]

Jacob   Six is a perfect number. Huh! …and, Arthur?

Arthur   28.

[Recall that 28 was the number introduced in the original problem.]

Jacob   28 is a perfect number. Mmmmmm.

Jacob   David, would you like to modify the *Hankins Hypothesis*?

David   Uh, they have to be perfect numbers, not just any number.

Jacob   Let's see here…So the sum of the factors of prime and composite *[Reads from board as he adjusts the 'Hankins Hypothesis']…* sum of the reciprocals of prime and composite factors of a perfect number will be one.

The sequence concluded with Jacob challenging the students to find the next perfect number and to see if it fit the newly revised *Hankins Hypothesis*.

Summary of the inductive teaching/learning episode. Although classic triadic exchange structure and univocal discourse were clearly evident within the teaching/learning episode described (especially in sequences 1 – 3), there were also indicators that mathematical meaning-making took place (especially in sequence 4). Common language related to the initial problem was developed in the first three sequences. This mutually built understanding served as a foundation for setting up exploration of richer concepts. Within sequence four, conjectures led to a preliminary "hypothesis" that Jacob named after the student who demonstrated the initial problem's solution—that is, *the sum of reciprocals of prime and composite factors of a number will always be one*. Jacob used discourse to model metacognitive processes ("… That's sort surprising that it would actually be one. I wonder if that's always true. I'm going to try another …") and then proceeded to have students explore possibilities ("… We've seen two examples now where it works. I'm sort of surprised … I don't know why it would work, but it seems to work… Would you guys check it out? Would you each take some other number and check it to see if, in fact, it does work?"), including corollaries to the theorem ("… doesn't work for **prime numbers** … So, let's see, we'll call this 'The Vargas Corollary.'"). Jacob's knowledge of mathematical content and pedagogy (Shulman, 2000) were instrumental in the orchestration of the discourse. He encouraged the students to explore and to conjecture, but also supplied meaningful verbal assessments that provoked them to generalize, to question, to justify, to reformulate, and to develop new meaning, thus including tendencies toward dialogic discourse.

It is important to note that the whole group discourse that was identified as promoting meaning-making did not stand on its own—it was integrally connected to earlier instances of both whole group and small group discussions. For example, recursive cycles of whole group and small group discussion were used to establish a rich problem as a frame of reference, develop common language, explore the problem, and develop and test hypotheses. The whole group meaning-making discourse explored connections between the problem's solution and other problems, built revised frames of reference, and demonstrated students' understanding (related to the original problem and revised hypotheses). Although the whole group discourse included classic triadic discourse structures, it also included GA and accountable talk. Furthermore, opportunities for rich verbal interactions within whole group discussion were built from preceding sequences that established a frame of reference, developed common language, and provided opportunities for exploration and conjecture. This case provides examples of how whole group discourse can be used to mediate student understanding of mathematics.

Final Remarks

> "Mathematics when it is finished, complete, all done, then it consists of proofs. But, when it is discovered, it always starts with a guess…" (Pólya, 1966)

In the Mathematics Association of America's video classic, "Let Us Teach Guessing," George Pólya, noted mathematician, mathematics educator, and problem-solving expert, can be seen teaching "guessing" to a group of university students. He begins the lesson with a rich problem that is unfamiliar to the students. As the lesson progresses, Pólya guides the students through a cycle of guesses, investigations, hypotheses, further investigations, and conjectures. Although no formal proofs are presented, evidence builds toward mathematical sense-making. Pólya makes use of triadic exchanges to facilitate the lesson, but does so with an art that seems to derive from deep understanding of both content and pedagogy. Jacob, a teacher with 35-years of experience in both mathematics content and pedagogy, seemed to make similar use of triadic structures as he worked through his own inductive teaching/learning cycle with his students. In the episode described, triadic structures seemed related to meaning-making when the verbal exchanges were connected to rich mathematical problems and to building (rather than simply conveying) students' understanding.

In revisiting previously stated concerns that triadic exchange structure may promote "illusory participation" (Lemke, 1990), we are reminded that consistent findings of classroom studies show that most U.S. teachers do tend to *state* information rather than develop ideas with their students (NCES, 1999, 2001). However, current research provides evidence that simply engaging students more actively in classroom discourse is not a panacea for improving mathematical achievement. For instance, Nathan and Knuth (2003) investigated a sixth-grade teacher's attempts to become more reform-oriented by working to decrease her authority-role in discussions while increasing her students' participation. Although the teacher was able to accomplish her discursive goals, "When the teacher elected to move away from her analytic role, the team [of researchers] observed that there was nothing added to the classroom culture to fill this gap in the discourse when there were major oversights, or when conflicting views among students arose" (p. 200). One striking example was when the students opted to "vote" on a mathematical concept. Lobato, Clarke, and Ellis (2005) suggest that although many reform-oriented teachers have downplayed "actions centered on introducing new mathematical ideas" (p. 104), that it may be appropriate to reconsider "telling" as a "system of actions," as long as the teacher focuses attention on the "development of the students' mathematics rather than on the communication of the teacher's mathematics" (p. 109). Indeed, perhaps a key piece is the professional judgment of an experienced teacher to know when to shift roles—that is, when to “step in” as a participant and when to “step out” to become a commentator of rules, norms, and concepts (Rittenhouse, 1998).

Although triadic exchange structure is often used to control instruction rather than to promote meaning-making, the episode from Jacob’s mathematics class described in this paper provided evidence that these structures can also be part of an inductive model of teaching. In his teaching, Jacob did include some judicious “telling” (Lobato et al., 2005), but also used his pedagogical content knowledge to judge when to “step in” and when to “step out” (Rittenhouse, 1998) in order to focus attention on the students’ mathematical meaning-making. And, like George Pólya (1966), Jacob invited students to guess, to hypothesize, to justify, and to make sense of mathematics. In summary, it is important to continue to explore all avenues of instruction that may invite students to make sense of mathematics. Even with its limitations, triadic exchange structure may still be used effectively by teachers whose learning/teaching goals are based on building students’ understanding, rather than on simply conveying the teacher’s ideas.

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Appendix—Glossary of Terms (as used in this paper)

Accountable Talk[3]	Classroom talk where participants are accountable to accurate and appropriate knowledge, to rigorous standards of reasoning, and to the learning community (Institute for Learning, 2001; Resnick, 1999).
Assessment (verbal)	For the purposes of this research, includes verbal moves (usually by the teacher) that help the teacher guide instruction and/or enhance learning.
Dialogic Discourse	Dialogue used to generate meaning (Knuth & Peressini, 2001; Lotman, 1988).
Discourse	"Purposeful talk on a mathematics subject in which there are genuine contributions and interaction" (Pirie, 1998; Pirie & Schwarzenberger, 1988, p. 460),
Episode	Largest unit in classroom discourse: all the talk to carry out a single activity
Exchange	Two or more moves occurring between speakers—typically structured as initiation, response, (and often) follow-up (IRF).
Exploratory Talk	"Speaking without answers fully intact (Cazden, 2001, p. 170). Exploratory talk is analogous to first drafts in writing, that is, first steps toward fluent and elaborated talk (Barnes, 1992).
Generative Assessment [GA]	Assessment that mediates discourse to promote students' active monitoring and regulation of thinking (i.e., metacognition) about the mathematics being taught (e.g., involvement that provokes elaboration and reflection, like, "What do you think?" or "Why do you think that?"). GA often changes the flow and function of the discourse.
Inert Assessment [IA]	Assessment that is likely to keep the flow and function of the discourse relatively constant (i.e., status quo), tending toward univocal (e.g., comments like, "Nice job," or, "That is not correct"). *Inert assessment* is based on Alfred North Whitehead's description of "inert ideas," that is, ones "that are merely received into the mind without being utilised, or tested, or thrown into fresh combinations" (Whitehead, 1964, p. 13).
IRF	Initiation, response, follow-up (Coulthard & Montgomery, 1981)
Leading Talk	Classroom talk in which the teacher controls the verbal exchanges, leading students toward the teacher's understanding. Although verbal exchanges occur, leading talk serves essentially the same purpose as monologic talk. Students' responses that have been *led toward* the teacher's intent are coded as leading talk.
Monologic Talk	Classroom talk in which one person (usually the teacher) is the only speaker. No verbal response is expected.
Move	Smallest building block in classroom discourse
Sequence	Unit that includes a single nuclear exchange with any exchanges that are bound to it (Wells, 1999).
Sequence Map	A graphic map showing all forms of talk and assessment contained within a single sequence and the flow between them
Triadic Structure	The most common structure of classroom discourse is *triadic dialogue* that includes IRE/IRF (initiation, response, evaluation/follow-up) (Cazden, 1988, 2001; Knuth & Peressini, 2001).
Univocal Discourse	One-way transmission of meaning (Knuth & Peressini, 2001; Lotman, 1988).
Zone of Proximal Development [ZPD]	"The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

[3] *Accountable Talk*[SM] is a service mark of the Institute for Learning Research and Development Center at the University of Pittsburgh (Institute for Learning, 2001).